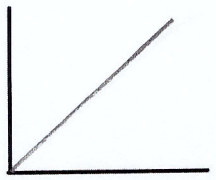
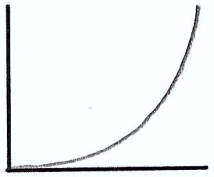
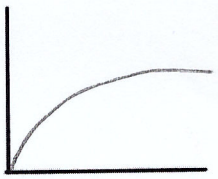
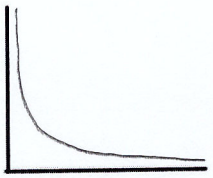
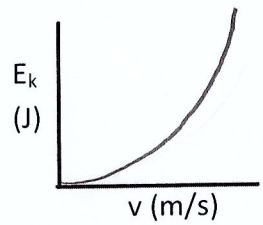
Curve Straightening & Understanding Meaning of Graphs

- It is an important skill to be able to manipulate and analysis graphs in order to fully understand all the information contained in the graphs
- Below is a chart that summarizes some common shaped graphs

* know shapes of graphs!

Line Shape	Equation	Relationship	Graph
linear	$y = mx + b$	directly proportional	
parabolic (around the y-axis)	$y = x^2$	proportional to the square	
inverse or parabolic (around the x-axis)	$y = \sqrt{x}$	proportional to the square root	
hyperbolic	$y = \frac{1}{x}$ or $y = \frac{1}{x^2}$	inversely proportional	

- EXAMPLE: Sketch the shape of the graph below.



$E_k = \frac{1}{2} m v^2$

$E_k \propto v^2$

- It is usually easier to analyze a linear graph, therefore, some curved graphs will need to be straightened

- Below are the steps needed for curve/graph straightening

  Step 1: Identify what is the manipulated variable (x) and what is the responding variable (y)

  Step 2: Identify or create the physics equation that describes the relationship between the two variables ← always plot "y" as is

  Step 3: Rearrange the equation to isolate the responding variable (y)

  Step 4: Recognize what is happening to the manipulated variable (ie. is it squared or inversed or square rooted) according to the physics equation and plot a new graph with the new manipulated variables as described by the physics equation. * only x-variables will be plotted different! *

- Straight line graphs can be interpreted and analyzed according to the mathematical formula that describes all straight line graphs; $y = mx + b$

- Below are the steps needed to interpret and analyze a straight line graph

  Step 1: Identify what is the manipulated variable (x) and what is the responding variable (y)

  Step 2: Identify or create the physics equation that describes the relationship between the x- and the y-variables

  Don't rearrange for the variable you are trying to solve! ← * **Step 3**: Rearrange the equation to isolate the responding variable (y)

  Step 4: Identify where the manipulated variable (x) is in the formula

  Step 5: Any part of the physics formula that is separated by an add or subtract function (+/-) and IS NOT multiplied by the manipulated variable (x), represent the y-intercept (b) of the graph. The y-intercept can be made up of one variable or several variables. If the physics formula has no add/subtract function, then the y-intercept is at zero (b=0).

  Step 6: Any variables left in the equation that have not been accounted for as either the x-variable, y-variable, or y-intercept, are grouped together to represent the slope (m) of the graph.

EXAMPLE: A student performs an experiment to measure how the centripetal force changes as the velocity is altered. He collects the data shown on the table below.

F_c (N) (y)	v (m/s) (x)	v^2 (m^2/s^2)
20.0	1.0	1.0
80.0	2.0	4.0
180	3.0	9.0
320	4.0	16.0
500	5.0	25.0

a. Identify the manipulating (x) and responding (y) variable. Plot the data on a graph.

b. Identify the physics equation that relates the two variables.

$F_c = ma_c$ but $a_c = \frac{v^2}{r}$ $\therefore$ $F_c = \frac{mv^2}{r}$

c. What needs to be plotted on the x-axis to produce a straight line graph? Re-plot this new, straight line graph.

$y = F_c$

$x = v^2$

d. Identify the meaning of the slope of this graph.

$F_c = \frac{mv^2}{r}$ needs to "fit into" $y = mx + b$ b/c it produces a straight line graph.

$y = m x + b$

$F_c = \left(\frac{m}{r}\right)(v^2) + 0$

$\therefore$ slope $= \frac{m}{r}$

e. Use your graph to determine the mass of the object if the radius of the circle is 1.15 m.

slope $= \frac{m}{r}$ $\Rightarrow$ $m = \text{slope}(r)$

$m = \left(20 \frac{N}{m^2/s^2}\right)(1.15 m)$

$m = 23 \frac{N \cdot m}{m^2/s^2}$

$m = 23$ kg

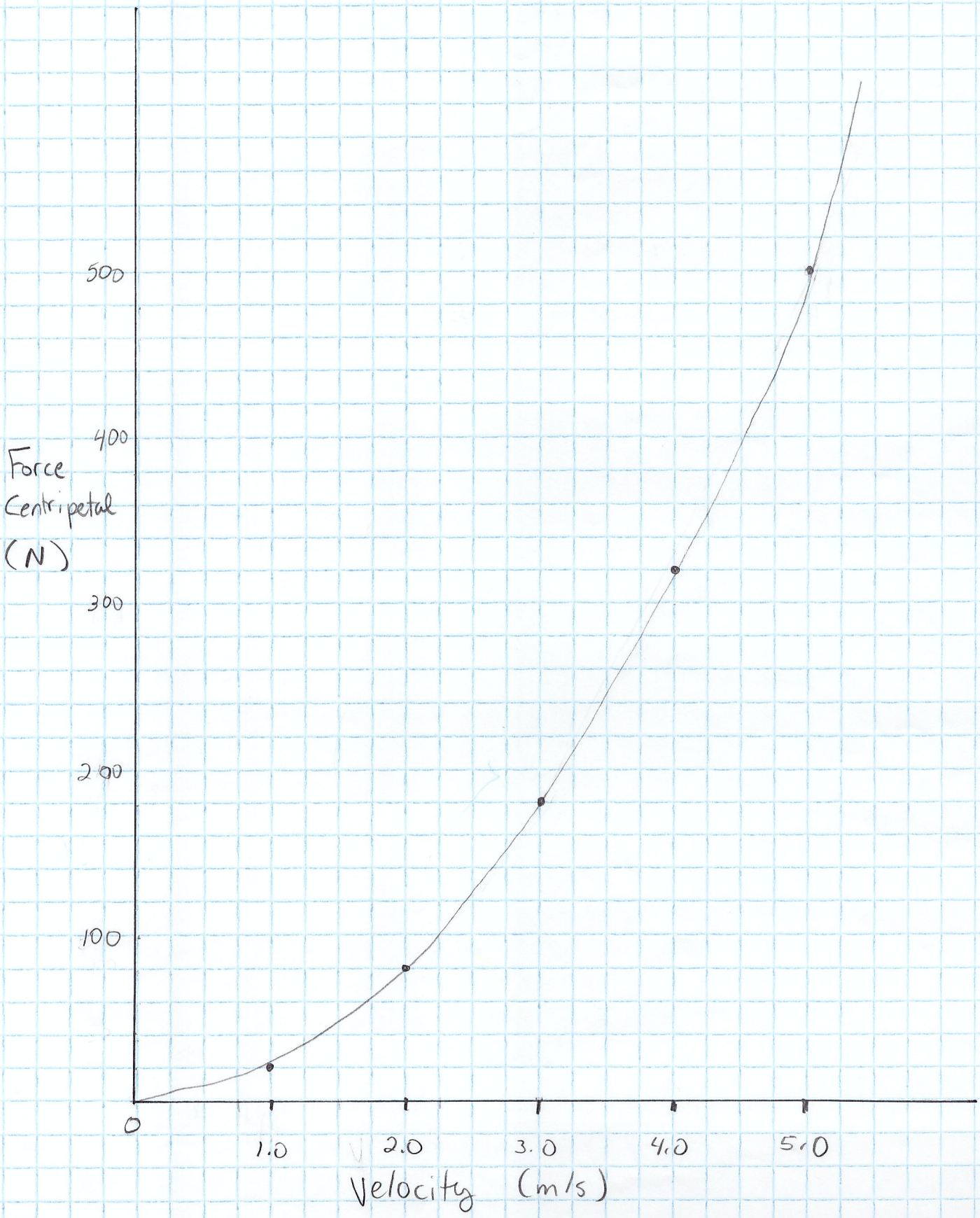
unit analysis

$\frac{N \cdot \cancel{m}}{\cancel{m^2}/s^2} = \frac{N}{m/s^2} = kg$

but $\frac{F_{net}}{m} = a$ $\therefore$ $\frac{F_{net}}{a} = m$

$\frac{N}{m/s^2} = kg$

Force Centripetal as Velocity is Altered.

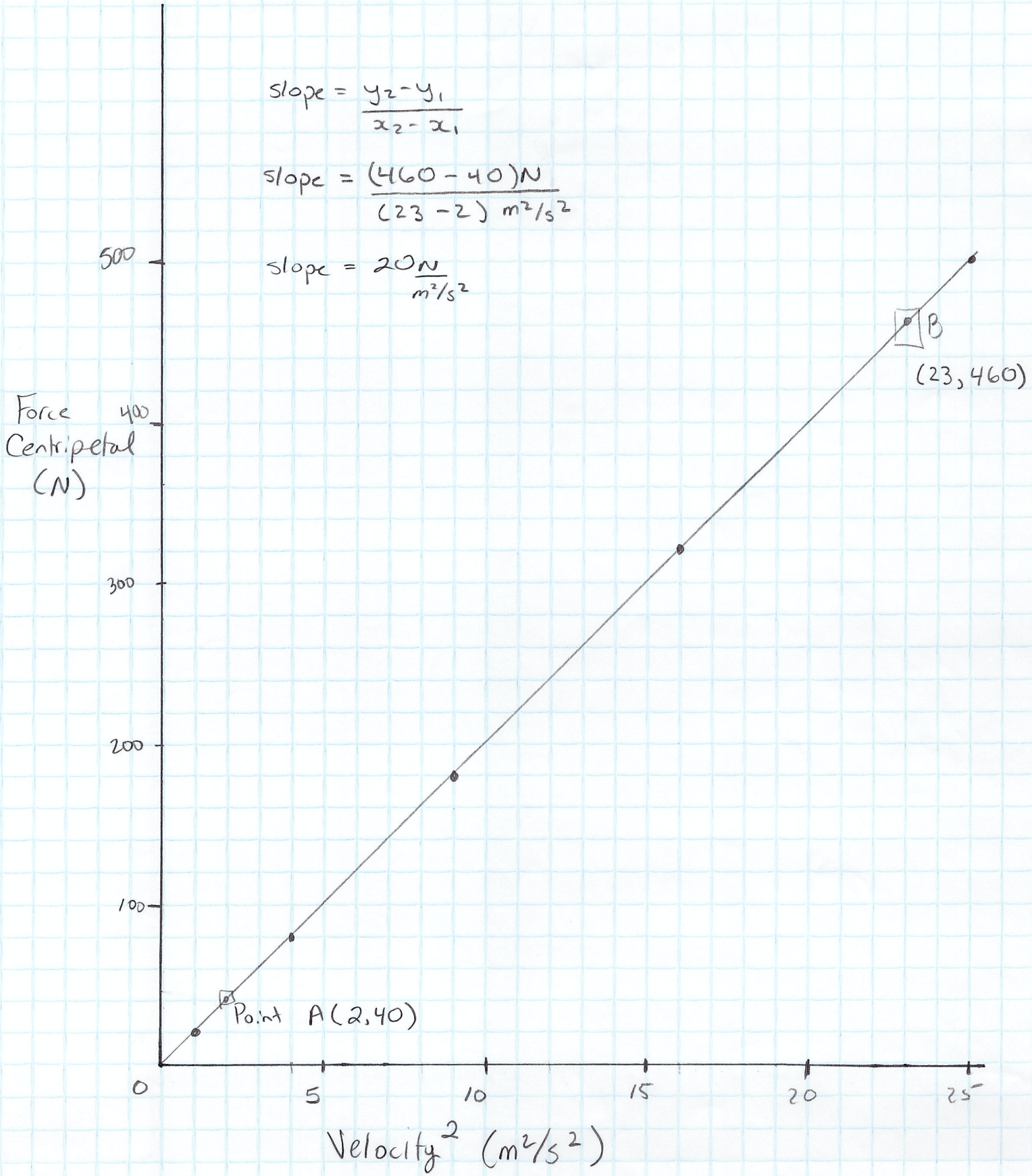


Force Centripetal as Velocity is Altered.

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\text{slope} = \frac{(460 - 40)\,N}{(23 - 2)\,m^2/s^2}$$

$$\text{slope} = 20\,\frac{N}{m^2/s^2}$$



EXAMPLE: During a NASA test, the velocity of the rocket is measured at different heights from the ground as it is being launched. The following information was recorded.

do on calculator!

Displacement (m) x	Velocity (m/s) y
25	37.4
50	53.0
75	64.7
100	74.8
125	84.0
150	91.6
175	99.0
200	105.0

x y

a. Identify the manipulating and responding variable.

b. Determine the equation that is used to describe this situation. If you plotted the x- and y-variables as is, sketch the shape of graph you would expect to see.

* acceleration problem!

$a = ?$

$v_i = 0 \text{ m/s}$

$d = x$

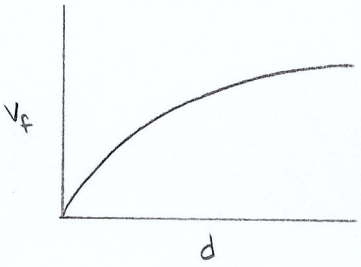
$v_f = y$

$\therefore \quad v_f^2 = \cancel{v_i^2}^{0} + 2ad$

$v_f = \sqrt{2ad}$

always isolate "y" variable

$v_f \propto \sqrt{d} \quad \therefore$



c. Identify what variables need to be plotted to produce a straight line graph. Plot this data.

b/c $v_f = \sqrt{2ad}$

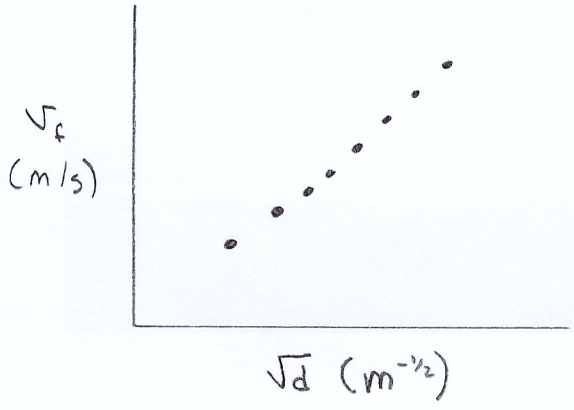
$y = v_f$

$x = \sqrt{d}$

graph on calculator

$x: [0, 15, 2]$

$y: [0, 110, 10]$



Linear Regression

$y = ax + b$

$a = 7.4392...$

$b = 0.3726...$

d. What does the slope of the linear graph represent?

$$v_f = \sqrt{2ad}$$

$$y = m x + b$$

$$v_f = (\sqrt{2a})(\sqrt{d}) + 0$$

$\therefore$ slope $= \sqrt{2a}$

e. Use your graph to determine what is the acceleration of the rocket?

slope $= \sqrt{2a}$

$$\therefore a = \frac{(\text{slope})^2}{2}$$

$$a = \frac{\left(7.4392\ldots \frac{m/s}{\sqrt{m}}\right)^2}{2}$$

$$a = 27.6709\ldots \frac{m^2/s^2}{m}$$

$a = 28 m/s^2$

*****Now try pg. 36 #4 (omit part e)*****

*****Complete Graphing Assignment*****