

Mass-Energy Equivalence

- Einstein concluded that there was a relationship between mass and energy and explained that mass could be converted into energy and vice-versa
 - This theory is known as the **mass-energy equivalence** and can be described by the following equation ↳ principle # 6

$$\Delta E = \Delta mc^2$$

where ΔE is the change in energy (J)
 Δm is the change in mass (kg)
 c is the speed of light ($3.0 \times 10^8 \text{ m/s}$)

EXAMPLE: What is the energy equivalence of a neutron at rest?

$E = ?$

$$m = 1.67 \times 10^{-27} \text{ kg}$$

$$\Delta E = \Delta mc^2$$

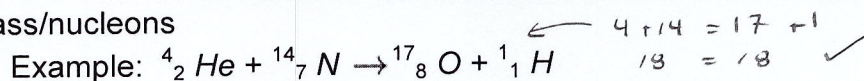
$$\Delta E = (1.67 \times 10^{-27} \text{ kg} - 0.0 \text{ kg}) (3.0 \times 10^8 \text{ m/s})^2$$

$$\Delta E = 1.503 \times 10^{-10} \text{ J}$$

$$\Delta E = 1.50 \times 10^{-10} \text{ J}$$

↳ has no kinetic energy

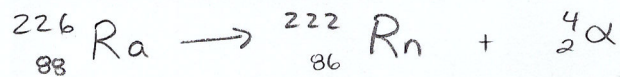
- The theory of mass-energy equivalence is used to help explain why nuclear reactions, such as fission and fusion, release so much more energy than a chemical reaction
- Recall that all nuclear reactions are described by the conservation of mass/nucleons



- When using standard nuclear notation, ${}_Z^AX$, the units for atomic mass (A) are in atomic mass units (u).
- $1 \text{ u} = 1.66 \times 10^{-27} \text{ kg}$ on data sheet!
- When using atomic mass numbers in the conservation of nucleons, these values are rounded the nearest whole number
- * On a smaller and more accurate scale, the total mass of all reactants is always greater than the total mass of all products in all nuclear reactions
- This difference in mass is known as the **mass defect (Δm)** and it is this "missing mass" that is converted into energy according to the mass-energy equivalence theory, explaining why nuclear reactions can release so much energy

EXAMPLE: Radium-226 will undergo alpha decay.

a. Write out this nuclear reaction.



b. The atomic masses are 226.025410u for radium-226, 222.017578u for radon-222, and 4.002603u for an alpha particle. Calculate the mass defect.

$$\Delta m = ?$$

$$\Delta m = |m_{\text{prod.}} - m_{\text{react.}}|$$

$$\Delta m = |(222.017578\text{u} + 4.002603\text{u}) - 226.025410\text{u}|$$

$$\Delta m = |226.020181\text{u} - 226.025410\text{u}|$$

$$\Delta m = 0.005229\text{u} \times \left(\frac{1.66 \times 10^{-27}\text{kg}}{1\text{u}} \right) = 8.68014 \times 10^{-30}\text{kg}$$

$$\Delta m = 8.68 \times 10^{-30}\text{kg}$$

c. Calculate the energy released by this nuclear reaction.

$$\Delta E = ?$$

$$\Delta m = 8.68014 \times 10^{-30}\text{kg}$$

$$\Delta E = \Delta mc^2$$

$$\Delta E = (8.68014 \times 10^{-30}\text{kg})(3.0 \times 10^8\text{m/s})^2$$

$$\Delta E = 7.812126 \times 10^{-13}\text{J} \times \left(\frac{1\text{eV}}{1.6 \times 10^{-19}\text{J}} \right)$$

$$\Delta E = 4882578.75\text{eV}$$

$$\Delta E = 4.88 \times 10^6\text{eV} \text{ or } 4.88\text{MeV}$$

d. Calculate the energy released per nucleon for this nuclear reaction.

$$\frac{4882578.75\text{eV}}{226} = 21604.33... \text{eV/nucleon}$$

$$2.16 \times 10^4 \text{eV/nucleon}$$

- **Particle-antiparticle annihilation** occurs when two particles collide, causing the total destruction of each particle and transforming all of their mass into energy according to the theory of mass-energy equivalence ($\Delta E = \Delta mc^2$)
 - Recall that the antiparticle has all the same characteristic and physical properties as the particle, but one physical property is the opposite

EXAMPLE: Calculate the energy that is produced when an electron-positron pair annihilate.

→ all mass converted into energy!

$$\Delta E = ?$$

$$\Delta m = m_f - m_i$$

$$\Delta m = m_e = 2(9.11 \times 10^{-31} \text{ kg})$$

$$\Delta m = 1.822 \times 10^{-30} \text{ kg}$$

$$\Delta E = \Delta m c^2$$

$$\Delta E = (1.822 \times 10^{-30} \text{ kg})(3.0 \times 10^8 \text{ m/s})^2$$

$$\Delta E = 1.6398 \times 10^{-13} \text{ J}$$

$$\Delta E = 1.64 \times 10^{-13} \text{ J}$$

Now try pg. 333 # 1, 2 & Practice Problems

Practice Problem

- Find the energy equivalence, in electron volts, for 0.221u. **[206 MeV]**
- An example of a fusion reaction is ${}^2_1\text{H} + {}^3_1\text{H} \rightarrow {}^4_2\text{He} + {}^1_0\text{n} + 17.6 \text{ MeV}$.
 - What is energy released per nucleon for this fusion reaction? **[3.52 MeV]**
 - Determine the amount of mass that is converted into energy for this fusion reaction. **[3.13x10⁻³⁰kg]**
- When a positron and an electron collide and undergo pair annihilation, two photons are produced. Explain why a single photon cannot be produced from the collision. **[Two photons need to be produced to maintain a conservation of momentum. The production a single photon would violate this law.]**
- When lithium-6 and hydrogen-2 undergo a nuclear reaction, two alpha particles are produced. Use the following data to calculate the energy released per nucleon for this nuclear reaction. **[2.80 MeV/nucleon]**

Isotope	Atomic Mass (u)
hydrogen-2	2.014102
helium-3	3.016029
helium-4	4.002603
lithium-6	6.015123

- Nitrogen-13 transmutes into carbon-13 by beta-positive decay. Calculate the energy released per nucleon for this beta-positive decay if the atomic masses are 13.005739u for nitrogen-13 and 13.003355u for carbon-13. **[2.75x10⁻¹³ J]**