Mass-Energy Equivalence

- Einstein concluded that there was a relationship between mass and energy and explained that mass could be converted into energy and vise-versa
  - This theory is known as the **mass-energy equivalence** and can be described by the following equation ↳ principle #6

$$\Delta E = \Delta mc^2$$

where ΔE is the change in energy (J)
Δm is the change in mass (kg)
c is the speed of light ($3.0\text{x}10^8$m/s)

EXAMPLE: What is the energy equivalence of a neutron at rest?
↳ has no kinetic energy

E = ?
$m = 1.67\text{x}10^{-27}$ kg

$\Delta E = \Delta mc^2$
$\Delta E = (1.67\text{x}10^{-27}\text{ kg} - 0.0\text{ kg})(3.0\text{x}10^8\text{ m/s})^2$
$\Delta E = 1.503 \text{x} 10^{-10}$ J
$\boxed{\Delta E = 1.50\text{x}10^{-10}\text{ J}}$

- The theory of mass-energy equivalence is used to help explain why nuclear reactions, such as fission and fusion, release so much more energy than a chemical reaction
- Recall that all nuclear reactions are described by the conservation of mass/nucleons

  Example: ${}^4_2 He + {}^{14}_7 N \rightarrow {}^{17}_8 O + {}^1_1 H$ ← 4 + 14 = 17 + 1
  18 = 18 ✓

  - When using standard nuclear notation, ${}^A_Z X$, the units for atomic mass (A) are in atomic mass units (u).
  - $1\text{u} = 1.66\text{x}10^{-27}$ kg on data sheet!
  - When using atomic mass numbers in the conservation of nucleons, these values are rounded the nearest whole number
  - * On a smaller and more accurate scale, the total mass of all reactants is always *greater* than the total mass of all products in all nuclear reactions
  - This difference in mass is known as the **mass defect (Δm)** and it is this "missing mass" that is converted into energy according to the mass-energy equivalence theory, explaining why nuclear reactions can release so much energy

EXAMPLE: Radium-226 will undergo alpha decay.

a. Write out this nuclear reaction.

$$^{226}_{88}Ra \longrightarrow {}^{222}_{86}Rn + {}^{4}_{2}\alpha$$

b. The atomic masses are 226.025410u for radium-226, 222.017578u for radon-222, and 4.002603u for an alpha particle. Calculate the mass defect.

$\Delta m = ?$

$\Delta m = |m_{prod.} - m_{react.}|$

$\Delta m = |(222.017578u + 4.002603u) - 226.025410u|$

$\Delta m = |226.020181u - 226.025410u|$

$\Delta m = 0.005229u \times \left(\frac{1.66 \times 10^{-27} kg}{1u}\right) = 8.68014 \times 10^{-30} kg$

$\boxed{\Delta m = 8.68 \times 10^{-30} kg}$

c. Calculate the energy released by this nuclear reaction.

$\Delta E = ?$

$\Delta m = 8.68014 \times 10^{-30} kg$

$\Delta E = \Delta m c^2$

$\Delta E = (8.68014 \times 10^{-30} kg)(3.0 \times 10^8 m/s)^2$

$\Delta E = 7.812126 \times 10^{-13} J \times \left(\frac{1eV}{1.6 \times 10^{-19} J}\right)$

$\Delta E = 4882578.75 eV$

$\boxed{\Delta E = 4.88 \times 10^6 eV \text{ or } 4.88 MeV}$

d. Calculate the energy released per nucleon for this nuclear reaction.

$\frac{4882578.75 eV}{226} = 21604.33... eV/nucleon$

$\boxed{2.16 \times 10^4 eV/nucleon}$

- **Particle-antiparticle annihilation** occurs when two particles collide, causing the total destruction of each particle and transforming all of their mass into energy according to the theory of mass-energy equivalence ($\Delta E = \Delta mc^2$)
  - Recall that the antiparticle has all the same characteristic and physical properties as the particle, but one physical property is the opposite

EXAMPLE: Calculate the energy that is produced when an electron-positron pair annihilate.
↳ all mass converted into energy!

$\Delta E = ?$

$\Delta m = \cancel{m_f}^{\,0} - m_i$

$\Delta m = m_i = 2(9.11 \times 10^{-31} kg)$

$\Delta m = 1.822 \times 10^{-30} kg$

$\Delta E = \Delta m c^2$

$\Delta E = (1.822 \times 10^{-30} kg)(3.0 \times 10^{8} m/s)^2$

$\Delta E = 1.6398 \times 10^{-13} J$

$\boxed{\Delta E = 1.64 \times 10^{-13} J}$

*****Now try pg. 333 # 1, 2 &Practice Problems*****

Practice Problem

1. Find the energy equivalence, in electron volts, for 0.221u. **[206 MeV]**

2. An example of a fusion reaction is ${}^{2}_{1}H + {}^{3}_{1}H \rightarrow {}^{4}_{2}He + {}^{1}_{0}n + 17.6 MeV$.
   a. What is energy released per nucleon for this fusion reaction? **[3.52 MeV]**
   b. Determine the amount of mass that is converted into energy for this fusion reaction. **[3.13×10^{-30}kg]**

3. When a positron and an electron collide and undergo pair annihilation, two photons are produced. Explain why a single photon cannot be produced from the collision. **[Two photons need to be produced to maintain a conservation of momentum. The production a single photon would violate this law.]**

4. When lithium-6 and hydrogen-2 undergo a nuclear reaction, two alpha particles are produced. Use the following data to calculate the energy released per nucleon for this nuclear reaction. **[2.80 MeV/nucleon]**

Isotope	Atomic Mass (u)
hydrogen-2	2.014102
helium-3	3.016029
helium-4	4.002603
lithium-6	6.015123

5. Nitrogen-13 transmutes into carbon-13 by beta-positive decay. Calculate the energy released per nucleon for this beta-positive decay if the atomic masses are 13.005739u for nitrogen-13 and 13.003355u for carbon-13. **[2.75×10^{-13} J]**