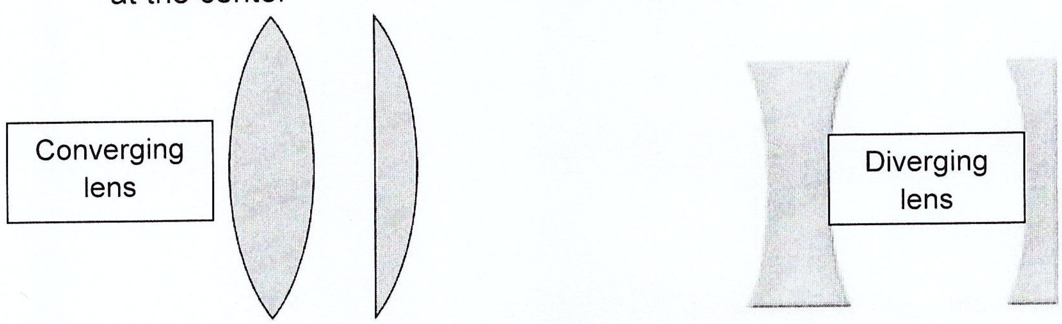
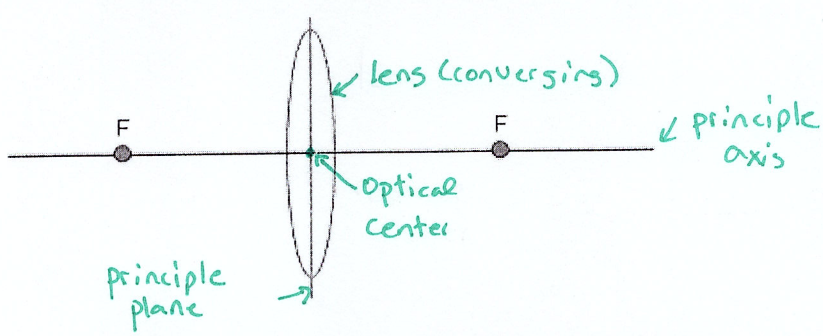
# Lenses

- For an object to be considered a lens, it must be made of a transparent material that has an index of refraction higher than air, therefore causing light to refract/bend to a common focus point as it passes through the lens

- All lenses are broken into two broad groups, depending on whether they focus light at a point (**converging** lens) or spread light out (**diverging** lens)
    - *Converging lenses* are shaped so that the lens is thicker at the center than at the edges
    - *Diverging lenses* are shaped so that the lens is thicker at the edges than at the center




- When completing *ray diagrams* for lenses (a drawing used to determine the characteristics of the image produced by a lens), the drawing needs to include a **principle axis** and a **principle plane**
    - The **principal axis** is an imaginary line that is perpendicular to the lens and cuts the length of the lens in half
    - The **principle plane** is a line that is perpendicular to the principle axis and runs length wise through the middle of the lens. We assume all refraction happens when the light reaches this principle plane.



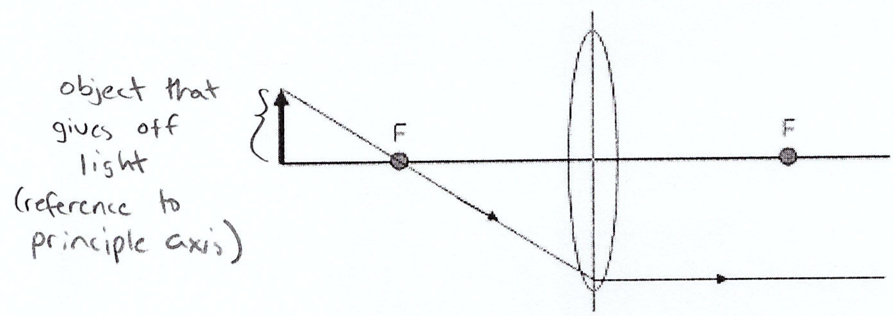

    - The **focal points (F)** are drawn on both sides of the lenses and are defined as the point where all parallel light rays will meet after passing through a lens

    - The **optical center** is the center of the lens

- When an object is placed in front of a lens, the image produced can have different characteristics.
    - Size: smaller, larger, or same size
    - Orientation: inverted or upright /erect
    - ✳ Image type: virtual or real; real images can be projected onto a screen, which means the images are on the opposite side of the lens in comparison to the object. Virtual images can be seen, but can't be projected onto a screen because the image and the object are on the same side of the lens.

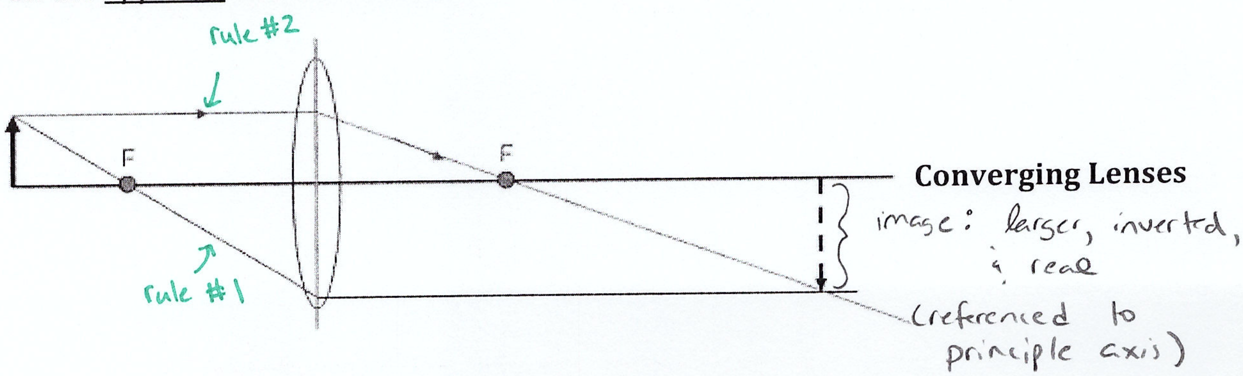
## Rules for Drawing Ray Diagrams for Lenses

- The image created by a lens is where all of the refracted rays meet after passing through the lens
- **Rule #1:** Any ray (ie. light) passing through the focal point of the lens will refract parallel to the principle axis. This rule doesn't apply to diverging lenses.

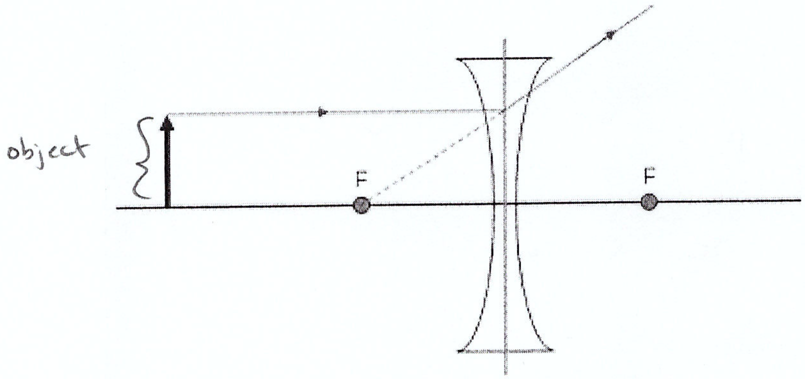



**Converging Lenses**

- **Rule #2:** Any ray parallel to the principle axis will refract so that is passes through the focal point. For a concave lenses, a ray will hit the lens and diverge/scatter. This means you need to extend the diverging ray to the focal point on the *opposite* side to the lens.

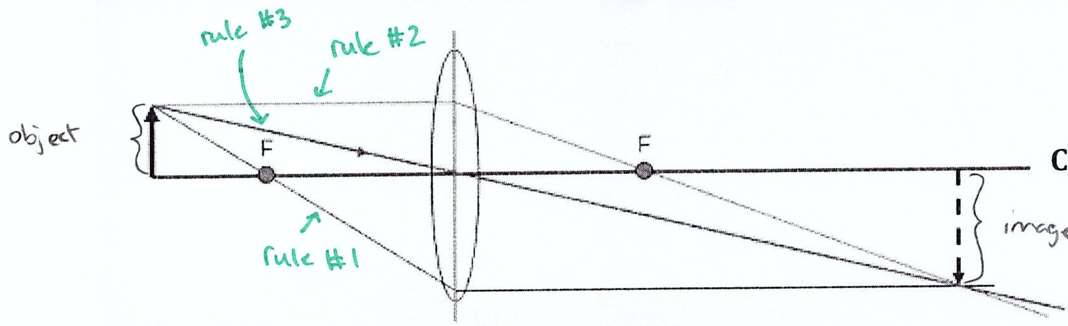



**Converging Lenses**

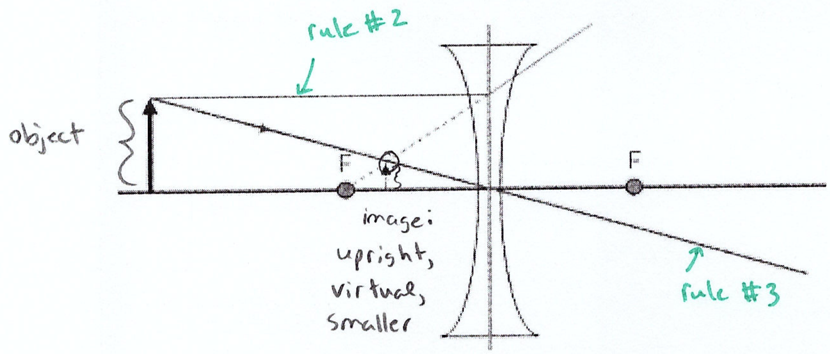



**Diverging Lenses**

- **<u>RULE #3:</u>** Any ray that passes through the optical center of the lens will pass through without any refraction. This rule applies to both types of lenses.

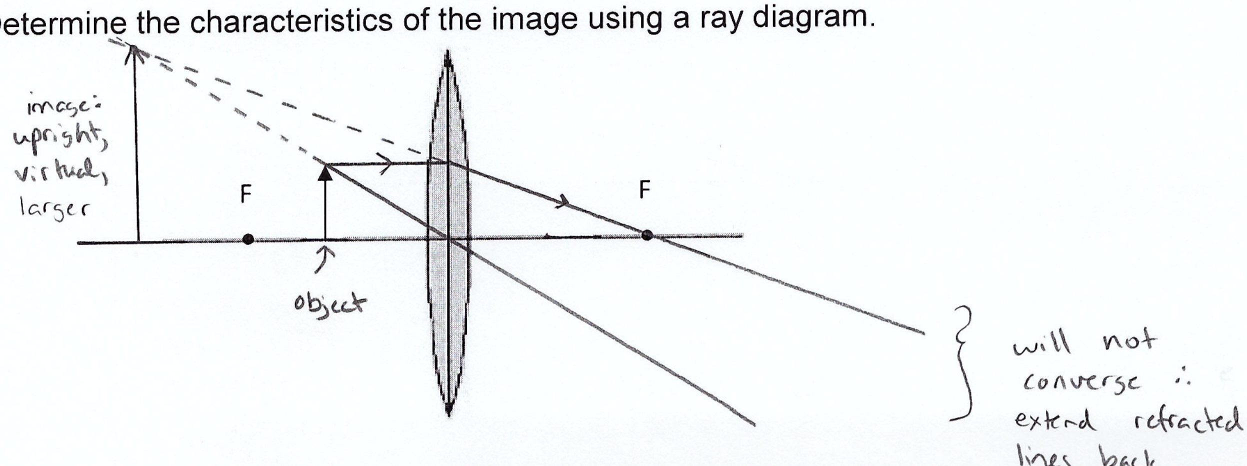



**Converging Lenses**




**Diverging Lenses**

<u>EXAMPLE:</u> Determine the characteristics of the image using a ray diagram.




*****Now try pg. 217 #1-3 (don't do the dimensions charts)*****

(for #2, a positive lens = converging lens)

- Lenses and the images produced by lenses can be described by two formulas.

$$\frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o} \quad \text{and} \quad m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

where

any units for distance as long as units are consistent

- $d_i$ is the distance of image from the principle plane of the lens
- $d_o$ is the distance of object from the principle plane of the lens
- f is the focal length/point (ie. the distance from the optical center to the focal point)
- $h_i$ is the height of image
- $h_o$ is the height of object

} in reference to principle axis

m is the magnification of image (no units)

* ○ A magnification (*m*) value larger than the absolute value of one (1) means the image is enlarged

* • Sign conventions

Very important to know!

| Variable | Positive (+) | Negative (-) |
|---|---|---|
| $d_o$ | Always positive | N/A |
| $d_i$ | Positive for real images (real images occur when the image and object are found on opposite sides of a lens because lenses allow light to pass through) | Negative for virtual images (virtual images occur when the image is found on the same side of the lens as the object) |
| f | Positive for converging lenses | Negative for diverging lenses |
| $h_i$ | Positive for upright images | Negative for inverted images |
| $h_o$ | Always positive | N/A |
| m | Positive for upright images | Negative for inverted images |

EXAMPLES: ↗ f is "+"

1. A converging lens has a focal length of 0.22m. If an object is placed 0.41m in front of the lens, identify the characteristics of the image that will be produced.

$f = +0.22m$

$d_o = +0.41m$

$$\frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o} \Rightarrow \frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o}$$

$$\frac{1}{d_i} = \frac{1}{+0.22m} - \frac{1}{+0.41m} = +2.106\ldots m^{-1}$$

$$\therefore d_i = \frac{1}{+2.106\ldots m^{-1}} = +0.4747\ldots m$$

↑ real image

$$m = \frac{-d_i}{d_o} = \frac{-(+0.4747\ldots m)}{+0.41m} = -1.157\ldots$$

inverted image

larger image b/c value is larger than 1

2. A glowing object 4.0cm tall is placed 2.0cm in front of a lens. If the height of the inverted image produced is 9.0cm, what is the distance between the object and its image?

$h_i$ is "–"

$h_o = +4.0cm$

$d_o = +2.0cm$

$h_i = -9.0cm$

$d = ? = d_o + d_i$

$$\frac{h_i}{h_o} = \frac{-d_i}{d_o} \Rightarrow d_i = \frac{-h_i d_o}{h_o}$$

$$d_i = \frac{-(-9.0cm)(+2.0cm)}{(+4.0cm)}$$

$d_i = +4.5cm$

↑ real image ∴ object & image on opposite sides of lens

$\therefore d_o = (+2.0cm) + (+4.5cm)$

$d_o = 6.5cm$

f is "−"

3. A diverging lens has a focal length of 20.0cm. Where should the object be placed in order for the virtual, upright image to be half as tall as the object?

$d_i$ is "−" ; m is "+" & $h_i$ is "+"

$f = -20.0cm$

$d_o = ?$

$m = +\frac{1}{2} = +0.5$

OR

$h_i = +\frac{1}{2}h_o = +0.5h_o$

$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$ (2)

$m = -\frac{d_i}{d_o}$ (1)

* use substitution of formulas

(1) $m = -\frac{d_i}{d_o} \Rightarrow d_i = -md_o$

(2) $\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$

$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{-md_o}$

* need lowest common denominator to add fractions!

$\frac{1}{f} = \frac{-m}{-md_o} + \frac{1}{-md_o}$

$\frac{1}{f} = \frac{-m+1}{-md_o}$

$-md_o = (-m+1)f$

$d_o = \frac{(-m+1)f}{-m} = \frac{(-0.5+1)(-20.0cm)}{-0.5}$

$d_o = +20.0cm$

*****Now try pg. 217 #7, 12 (acceptable), 4, 8 & P.P. 1, 2 (intermediate), pg. 220 #13, 14 & P.P. #2 (excellence)*****

**Convex lens = converging lens & concave lens = diverging lens**

# Practice Problems

1. An object is placed 17.0cm from a diverging lens. If the virtual, upright image is $1/5^{th}$ smaller than the real object, what is the focal length of the lens?
   **[-4.25 cm]**

2. An old camera allows light to reflect off the object it focusses on and then refracts the light through its lens onto a sensitive film. Consider a 1.95m tall person that is 4.50m from the camera. The inverted image that is exposed onto the film is 1.10cm tall. Determine the type and focal length of the lens used in the camera.
   **[diverging lens with focal length of 2.55 cm]**

3. A diverging lens has a focal length of 22.0cm and a magnification of 0.25. Calculate the distance from the lens at which the image will appear and the type of image it will be. **[16.5 cm, virtual]**