Compton Effect

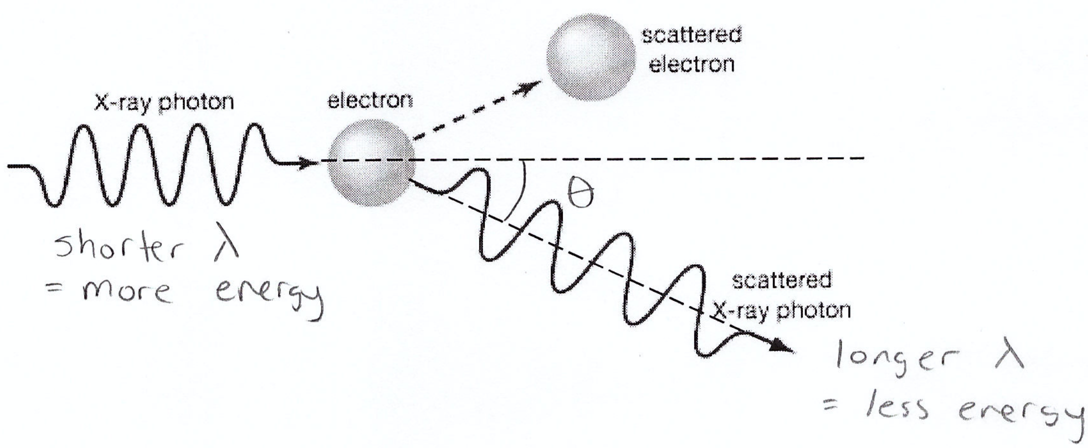
- Compton took the photoelectric effect experiment and made one modification; he used the high energy photon source of x-rays instead of shining light/UV light on a metal surface.
    - When high energy x-rays were directed at a metal surface, Compton still observed electrons being ejected from the metal surface but observed that the *x-rays scattered* after hitting the metal surface
    - The scattered x-ray had an *increase in wavelength* because the photon transferred energy to the electron upon colliding, but the photon also experienced a *change in direction*!
    - * Compton was able to show that the increase in wavelength and the change in direction of the scattered x-ray compared to the incident x-ray followed the conservation of momentum law!

principle # 4

$$\Sigma p_{before} = \Sigma p_{after}$$

$$p_{incident\ photon} = p_{scattered\ photon} + p_{scattered\ electron}$$

* this collision is elastic b/c at a molecular level!



- * Showing that photons had momentum was strong evidence of the particle model of EMR

principle # 5

- * The conversation of energy still applies to the Compton effect as it did with the photoelectric effect

$$E_{in} = E_{out}$$

$$E_{\substack{incident \\ x\text{-ray}}} = E_{\substack{scattered \\ x\text{-ray}}} + E_{k,e^-}$$

where

$$E_{x\text{-ray}} = \frac{hc}{\lambda}$$

$$E_{k,e^-} = \frac{1}{2}mv^2$$

* the work function (W) is so small compared to the other energies that it is simply ignored!

- Since EMR has no rest mass, the classical method for calculating momentum (p=mv) could **not be** used
  - A new formula was derived to calculate the momentum of a photon, based on the wavelength of the EMR

$$p = \frac{h}{\lambda}$$

where p is momentum of a photon (kg·m/s)
h is Planck's constant (needs to be 6.63×10^{-34} J·s)
λ is wavelength (m)

- Again, notice how a formula for momentum, which supports the particle theory, still contains ideas from the wave theory (ie. wavelength). This continues to support the *particle-wave duality*; EMR acts as both a wave and a particle simultaneously ↳ principle #9

- Compton was able to create a formula that could calculate the change in wavelength of the x-ray after being directed at a metal surface.

$$\Delta\lambda = \frac{h}{mc}(1 - \cos\theta)$$

where Δλ is a change in wavelength ($\lambda_f - \lambda_i$) (m)
h is Planck's constant (need to use 6.63×10^{-34} J·s)
m is mass of an electron (9.11×10^{-31} kg)
c is the speed of light (3.0×10^{8} m/s)
θ is the angle between the initial and final wavelengths
↳ not e⁻ !

- There is also an equation that relates the energy of a photon to its momentum

$$E = pc$$

where *E* is energy of the photon (J) - needs to be in joules
p is the momentum of the photon (kg·m/s)
c is the speed of light (3.0×10^{8} m/s)

EXAMPLES

1. Calculate the frequency of EMR whose photons have momentum of 2.80×10^{-27} kg·m/s each.

↳ cannot use $p = mv$!

$f = ?$

$p = 2.80 \times 10^{-27}\ kg \cdot m/s$

$v = \lambda f$ ②

↓

$p = \frac{h}{\lambda}$ ①

① $\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}\ J \cdot s}{2.80 \times 10^{-27}\ kg \cdot m/s} = 2.367857... \times 10^{-7}\ m$

② $f = \frac{v}{\lambda} = \frac{3.00 \times 10^{8}\ m/s}{2.367857... \times 10^{-7}\ m} = 1.2669... \times 10^{15}\ Hz$

$\boxed{f = 1.27 \times 10^{15}\ Hz}$

2. If a photon has an energy of 6.00eV, what is the momentum of the photon?

$E = 6.00eV \times \left(\frac{1.6 \times 10^{-19} J}{1 eV}\right)$

$E = 9.6 \times 10^{-19}\ J$

$p = ?$

$E = pc$

$p = \frac{E}{c} = \frac{9.6 \times 10^{-19}\ J}{3.0 \times 10^{8}\ m/s}$

$\boxed{p = 3.2 \times 10^{-27}\ kg \cdot m/s}$

unit analysis

$\frac{\cancel{J}}{m/s} \times \left(\frac{kg \cdot (m/s)^2}{\cancel{J}}\right)$

$= kg \cdot m/s$ ✓

$E_K = \frac{1}{2}mv^2$

$J = kg \cdot (m/s)^2$

3. An X-ray with a wavelength of 1.28×10^{-12} m is shot at a piece of graphite. The X-ray is observed to have scattered at an angle of 15.0° from the original path.
   a. Determine the wavelength of the scattered X-ray.
   b. Determine the energy gained by the ejected electron.

a.)

$\lambda_i = 1.28 \times 10^{-12}\, m$

$\theta = 15.0°$

$\lambda_s = ?$

$\Delta\lambda = \lambda_f - \lambda_i$ ②

$\downarrow$

$\Delta\lambda = \frac{h}{mc}(1 - \cos\theta)$ ①

① $\Delta\lambda = \frac{6.63 \times 10^{-34}\, J \cdot s}{(9.11 \times 10^{-31}\, kg)(3.0 \times 10^{8}\, m/s)} (1 - \cos(15.0°)) = 8.26607... \times 10^{-14}\, m$

② $\Delta\lambda = \lambda_s - \lambda_i \Rightarrow \lambda_s = \Delta\lambda + \lambda_i$

$\lambda_s = (8.26607... \times 10^{-14}\, m) + 1.28 \times 10^{-12}\, m = 1.36266... \times 10^{-12}\, m$

$\boxed{\lambda_s = 1.36 \times 10^{-12}\, m}$

b.) $E_{incident} = E_{scattered} + E_{k,e^-}$

$E_{k,e^-} = E_{incident} - E_{scattered}$

$E_{k,e^-} = \frac{hc}{\lambda_i} - \frac{hc}{\lambda_s} = hc\left(\frac{1}{\lambda_i} - \frac{1}{\lambda_s}\right)$

$E_{k,e^-} = (6.63 \times 10^{-34}\, J \cdot s)(3.0 \times 10^{8}\, m/s)\left(\frac{1}{1.28 \times 10^{-12}\, m} - \frac{1}{1.36266... \times 10^{-12}\, m}\right)$

$E_{k,e^-} = 9.426... \times 10^{-15}\, J$

$\boxed{E_{k,e^-} = 9.43 \times 10^{-15}\, J}$

4. An X-ray with a wavelength of 3.30×10^{-12} m is directed towards a piece of thin gold foil. The initial X-ray is travelling east and strikes an electron initially at rest. If the ejected electron is travelling at 1.07×10^{8} m/s to the east, what is the momentum of the scattered X-ray?

↳ vector quantity ∴ use conservation of momentum!

Before

$\lambda = 3.30 \times 10^{-12}$ m

$p = \frac{h}{\lambda}$

$p_p = \frac{6.63 \times 10^{-34} \text{ J·s}}{3.30 \times 10^{-12} \text{ m}}$

$p_p = 2.00909... \times 10^{-22}$ kg·m/s

e^-

$m = 9.11 \times 10^{-31}$ kg

$v = 0.0$ m/s

$p = mv$

$p_{e^-} = 0.0$ kg·m/s

After

?

$p'_p = ?$

e^-

$m = 9.11 \times 10^{-31}$ kg

$v = 1.07 \times 10^{8}$ m/s

$p = mv$

$p'_e = 9.7477 \times 10^{-23}$ kg·m/s

$$\Sigma p = \Sigma p'$$

$$2.00909... \times 10^{-22} + 0 = p'_p + 9.7477 \times 10^{-23}$$

$$1.03432... \times 10^{-22} = p'_p$$

$$\boxed{p'_p = 1.03 \times 10^{-22} \text{ kg·m/s, east}}$$

*****Now try pg. 270 #4-7 (acceptable), 10, 12, 14, 15, 20 (intermediate), pg. 277 #2, & Practice Problems (excellence)*****

Practice Problems

1. An X-ray with a wavelength of 1.60×10^{-11} m is directed towards a piece of thin gold foil. The initial X-ray is travelling east and strikes an electron initially at rest. If the scattered X-ray bounces straight back with a wavelength of 3.70×10^{-11} m,
    a. what is the momentum of the electron? **[5.94×10^{-23} kg·m/s, east]**
    b. what is the velocity of the electron? **[6.52×10^{7} m/s, east]**

2. An X-ray with a wavelength of 9.65×10^{-11} m is shot directly north at a piece of graphite. The scattered X-ray has a wavelength of 2.80×10^{-10} m and is heading 64.0°, E of S. Determine the velocity of the scattered electron. **[8.99×10^{6}, m/s 74.9° N of W]**