Two Dimensional Collisions

- *Total* momentum must be conserved in ***each*** dimension, therefore

$$\sum p_x = \sum p_x' \qquad \sum p_y = \sum p_y'$$

- Need to use trig. functions and Pythagoreans' theorem to find the *total* momentum before and after the interaction in the *x* and *y* directions **separately**

$$\cos\theta = \frac{adj}{hyp}, \quad \sin\theta = \frac{opp}{hyp}, \quad \tan\theta = \frac{opp}{adj}, \quad a^2 + b^2 = c^2$$

- The process is the same for solving 2-D collisions as the linear collisions, but you need to find p_x and p_y for each object and then find the $p_{x,\,total}$ (before and after)and $p_{y,total}$ (before and after)

Examples

1.) A 4.0kg object is travelling south at a velocity of 2.8m/s when it collides with a 6.0kg object travelling east at a velocity of 3.0m/s. If these two objects stick together upon collision, at what velocity does the combined masses move?

Before		After
①	②	①②
$m_1 = 4.0$	$m_2 = 6.0$	$m = 10$
$v_1 = -2.8$	$v_2 = +3.0$	$v = ?$
$P_x = 0$	$P_x = +18$	$P_x' = P_x'$
$P_y = -11.2$	$P_y = 0$	$P_y' = P_y'$

$\therefore$ $P_{x\,total} = P'_{x\,total}$

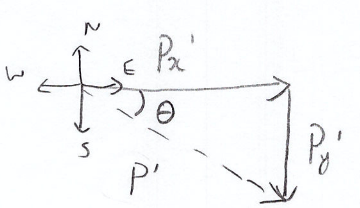
$0 + (+18) = P_x'$

$+18 = P_x'$

$P_{y\,total} = P_y'\,_{total}$

$-11.2 + 0 = P'_{y\,total}$

$-11.2 = P'_{y\,total}$



$P' = \sqrt{(-11.2)^2 + (18)^2}$

$P' = 21.2 \text{ kg}\cdot\text{m/s}$

$\theta = \tan^{-1}\left(\frac{11.2}{18}\right) = 31.89...$

$\therefore$ $v = \frac{P}{m} = \frac{21.2 \text{ kg}\cdot\text{m/s}}{10.0 \text{ kg}}$ $\Rightarrow$ **2.1 m/s, 32° S of E = v**

method #1 using trig!

2.) A 4.0kg object is moving at an unknown velocity when it collides with a 6.1kg stationary object. After the collision, the 6.1kg object is travelling at a velocity of 2.8m/s 32° N of E and the 4.0kg object is travelling at a velocity of 1.5m/s 41° S of E. What was the velocity of the 4.0kg object before the collision?

Before

? (1)

$m_1 = 4.0$

$v_1 = ?$

p_{x_1}

p_{y_1}

(2)

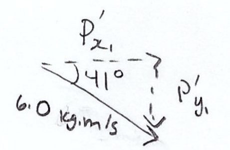
$m_2 = 6.1$

$v_2 = 0$

$p_{x_2} = 0$

$p_{y_2} = 0$

After



(1)

$m = 4.0$

$v'_1 = 1.5$

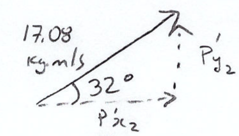
$p'_1 = 6.0$

$p'_{x_1} = \cos(41°)\,6.0$

$p'_{x_1} = +4.528$

$p'_{y_1} = \sin(41°)\,6.0$

$p'_{y_1} = -3.936...$



(2)

$m = 6.1$

$v'_2 = 2.8$

$p'_2 = 17.08$

$p'_{x_2} = \cos(32°)\,17.08$

$p'_{x_2} = +14.484...$

$p'_{y_2} = \sin(32°)\,17.08$

$p'_{y_2} = +9.05$

$\sum p_x = \sum p_x'$

$p_{x_1} + 0 = p_{x_1}' + p_{x_2}'$

$p_{x_1} = (4.528) + (14.484...)$

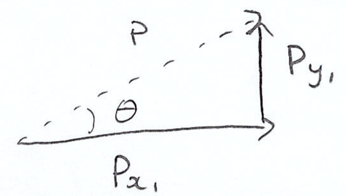
$p_{x_1} = 19.012...$

$\sum p_y = \sum p_y'$

$p_{y_1} + 0 = p'_{y_1} + p'_{y_2}$

$p_{y_1} = (-3.936...) + (9.05)$

$p_{y_1} = +5.114...$



$a^2 + b^2 = c^2$

$p = \sqrt{(19.012...)^2 + (5.114...)^2}$

$p = 19.688....\ kg{\cdot}m/s$

$\therefore v = \frac{p}{m} = \frac{19.688...\ kg{\cdot}m/s}{4.0\,kg} = 4.922...\,m/s$

$\theta = \tan^{-1}\left(\frac{p_{y_1}}{p_{x_1}}\right)$

$\theta = \tan^{-1}\left(\frac{5.114...}{19.012...}\right)$

$\theta = 15.056...°$

$v = 4.9\,m/s$, 15° N of E

Now try pg. 58 #1, 2 (intermediate), 3, 6, 8 (excellence)

Method # 2 using sin & cos laws!

2.) A 4.0kg object is moving at an unknown velocity when it collides with a 6.1kg stationary object. After the collision, the 6.1kg object is travelling at a velocity of 2.8m/s 32° N of E and the 4.0kg object is travelling at a velocity of 1.5m/s 41° S of E. What was the velocity of the 4.0kg object before the collision?

Before

?

(1)

$m_1 = 4.0\,kg$

$v_1 = ?$

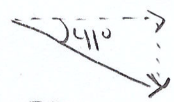
[2]

$m_2 = 6.1\,kg$

$V_2 = 0\,m/s$

$P_2 = 0\,m/s$

After

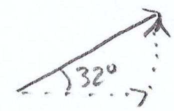


(1)

$m_1 = 4.0\,kg$

$v_1' = 1.5\,m/s$

$P_1' = 6.0$

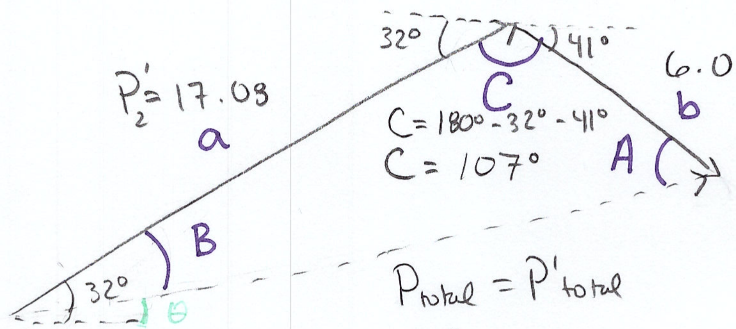


[2]

$m_2 = 6.1\,kg$

$v_2' = 2.8\,m/s$

$P_2' = 17.08$



$C = 180° - 32° - 41°$

$C = 107°$

$c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 = (17.08)^2 + (6.0)^2 - 2(17.08)(6.0)\cos(107°)$

$c^2 = 387.6509...$

$c = 19.6888...\ kg\cdot m/s$

$\frac{b}{\sin B} = \frac{c}{\sin C}$

$\sin B = \frac{b \sin C}{c}$

$B = \sin^{-1}\left(\frac{b \sin C}{c}\right)$

$B = \sin^{-1}\left(\frac{(6.0)\sin(107)}{19.6888...}\right)$

$B = 16.9433...°$

$\theta = 32° - 16.9433...°$

$\theta = 15.0566...°$

* momentum of m_1 before collision *

$p = mv \rightarrow v = \frac{p}{m} = \frac{19.6888...\ kg\cdot m/s}{4.0\,kg} = 4.9222...\ m/s$

$v_1 = 4.9\,m/s,\ 15°\ N\ of\ E$

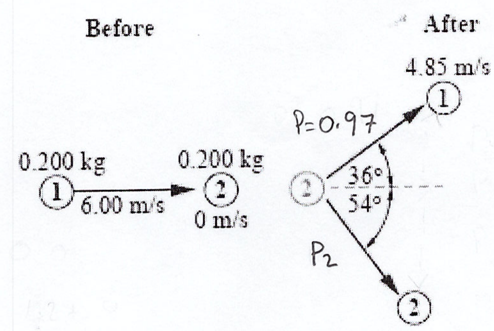
*****Now try pg. 58 #1, 2 (intermediate), 3, 6, 8 (excellence)*****

Method #1 using trig!

2-D Collisions – Review

Use the following information to answer the next question.

Two identical metal pucks were made to collide on a frictionless surface. Before the collision, puck 1 was moving at 6.00 m/s and puck 2 was stationary. After the collision, the pucks moved as shown in the diagram below.



The magnitude of the **momentum** of puck 2 after the collision was

A. 1.33 kg·m/s
B. 0.970 kg·m/s
(C.) 0.705 kg·m/s
D. 0.570 kg·m/s

$a^2 + b^2 = c^2$

$P = \sqrt{(-0.5701...)^2 + (0.4152...)^2}$

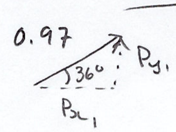
$P = 0.7053...$

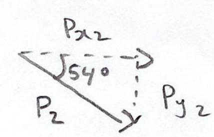
Before

$P_{y,total} = 0.0$

$P_{x,total} = 1.2 + 0 = 1.2$

After





?

$P_{x1} = \cos(36°)(0.97)$

$P_{x1} = 0.7847....$

$P_{y1} = \sin(36°)(0.97)$

$P_{y1} = 0.5701....$

$\Sigma P_y = \Sigma P_y'$

$0 = 0.5701... + P_{y2}$

$\therefore P_{y2} = -0.5701...$

$\Sigma P_x = \Sigma P_x'$

$1.2 = 0.7847... + P_{x2}$

$P_{x2} = 0.4152...$

method #2 using sin & cos laws!

2-D Collisions – Review

Use the following information to answer the next question.

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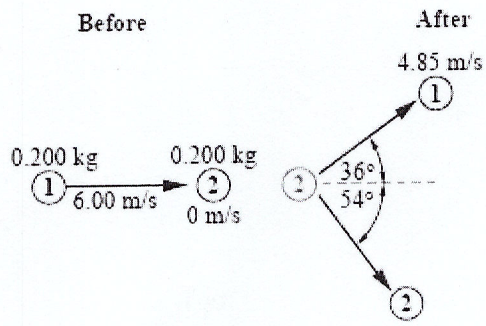

The magnitude of the **momentum** of puck 2 after the collision was

A. 1.33 kg·m/s

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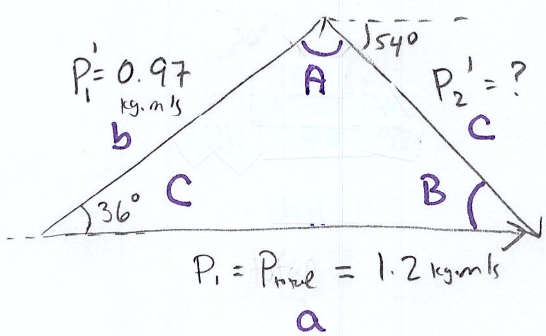


$c^2 = a^2 + b^2 - 2ab\cos C$

$(P_2')^2 = (0.97)^2 + (1.2)^2 - 2(0.97)(1.2)\cos(36°)$

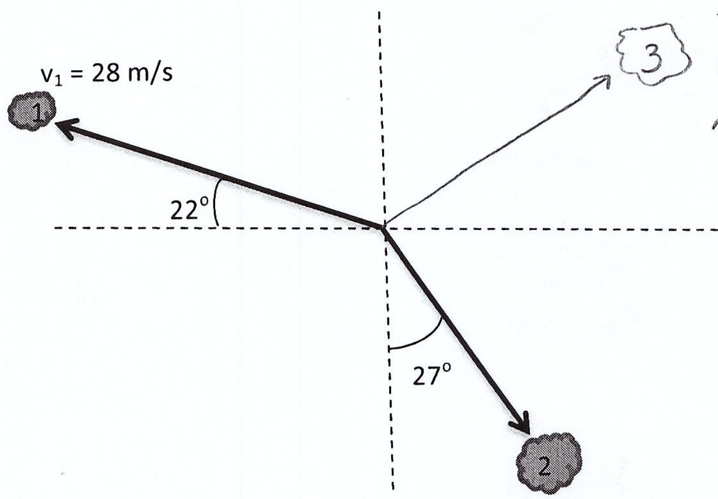
$(P_2')^2 = 0.497508...$

$P_2' = 0.7053...$ kg·m/s

method #1 using trig!

Taking 2-D Collisions One Step Farther

An object with a total mass of 15.7 kg is sitting at rest when it explodes into three pieces. One piece with a mass of 4.7 kg moves up and to the left at an angle of 22° with a speed of 28 m/s. A second piece with a mass of 5.2 kg moves down and to the right an angle of 27° at a speed of 20.7 m/s. What is the velocity of the third piece after the explosion?



$v_3 = ?$

$m_3 = 15.7kg - 4.7kg - 5.2kg$

$m_3 = 5.8kg$

Before

m = 15.7

v = 0.0

p = 0.0

$p_x = 0$

$p_y = 0$

After

$m_1 = 4.7$	$m_2 = 5.2$	$m_3 = 5.8$
$v_1 = 28$, 22° N of w	$v_2 = 20.7$, 27° E of S	v_3
$P_1 = 131.6$, 22° N of w	$P_2 = 107.64$, 27° E of S	P_3
$P_{x_1} = \cos(22°)131.6$ $P_{x_1} = -122.017...$	$P_{x_2} = \sin(27°)107.64$ $P_{x_2} = 48.867...$	$P_{x_3} = a$
$P_{y_1} = \sin(22°)\,131.6$ $P_{y_1} = 49.298...$	$P_{y_2} = \cos(27°)\,107.64$ $P_{y_2} = -95.907...$	$P_{y_3} = b$

$$\Sigma p_x = \Sigma p_x'$$

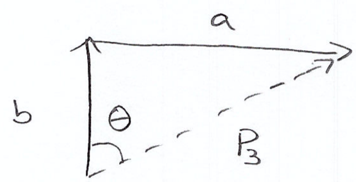
$$0 = (-122.017...) + (48.867...) + a$$

$$73.15... = a$$

$$\Sigma p_y = \Sigma p_y'$$

$$0 = (49.298...) + (-95.907...) + b$$

$$46.609... = b$$



$$\theta = \tan^{-1}\left(\frac{73.15....}{46.609...}\right) = 57.49°....$$

$$a^2 + b^2 = c^2$$

$$P_3 = \sqrt{(46.609...)^2 + (73.15...)^2}$$

$$P_3 = 86.737... \text{ kg}\cdot\text{m/s}$$

but $v = \frac{P}{m} = \frac{86.737... \text{ kg}\cdot\text{m/s}}{5.8 \text{ kg}} = 14.954... \text{ m/s}$

$v = 15$ m/s, 57° E of N

method #2 using cos & sin laws!

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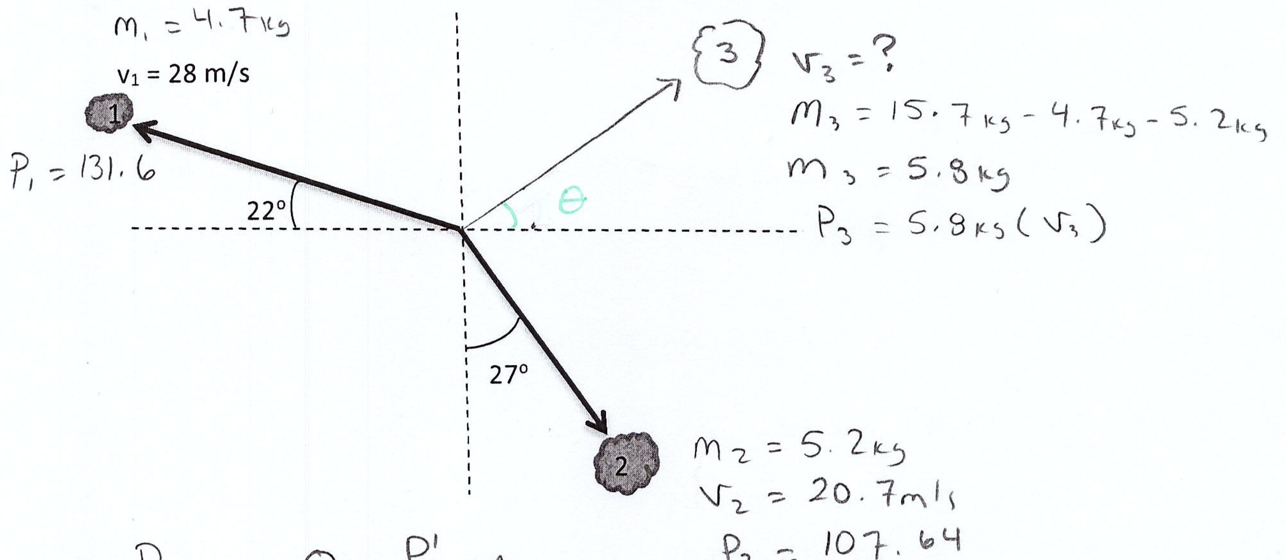
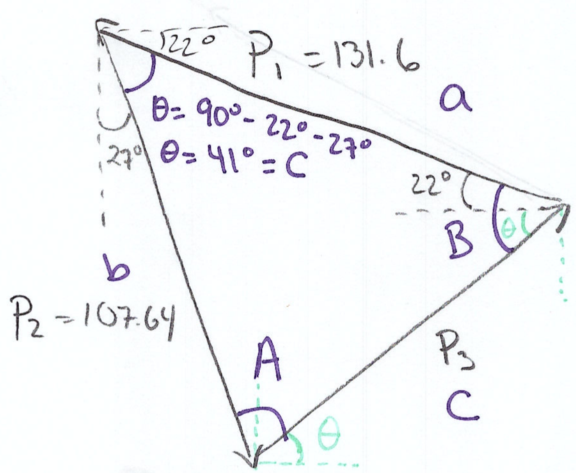


$P_{total} = 0 = P'_{total}$



$c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 = (131.6)^2 + (107.64)^2 - 2(131.6)(107.64)\cos(41°)$

$c^2 = 7523.367\ldots$

$c = 86.7373\ldots$ kg·m/s $= P_3$

$\therefore p = mv \rightarrow v = \frac{p}{m}$

$v = \frac{86.7373\ldots \text{ kg·m/s}}{5.8 \text{ kg}} = 14.9547\ldots$ m/s

$\frac{b}{\sin B} = \frac{c}{\sin C} \rightarrow \sin B = \frac{\sin C (b)}{c}$

$B = \sin^{-1}\left(\frac{\sin(41°)(107.64)}{86.7373\ldots}\right) = 54.5045\ldots°$

$\therefore \theta = 54.5045\ldots° - 22° = 32.504\ldots°$

$v = 15$ m/s, 33° N of E